Digital Filter Design,
Biomedical Signal Noise Reduction,
ε-insensitive Loss Function.

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A NEW CLASS OF DIGITAL FILTERS DESSIGNED FOR ECG NOISE REDUCTION

This paper describes a new method for design of linear phase finite impulse response (FIR) filters. This new approach, based on the ε-insensitive loss function, allows the design process to take into account not only constraints specified in the frequency domain, but also constraints on the output, time domain, signal. The performances of the proposed approach are shortly illustrated with a design of a highpass filter used for ECG baseline wander reduction.

1. INTRODUCTION

The first step in all ECG signal processing systems is baseline wander and powerline interference reduction. The baseline wander is caused by varying electrode-skin impedance, patient's move and breath and its frequency range is placed usually under 1.0 Hz [12, 2]. Generally, methods used to reduce this kind of disturbance can be divided into three groups: methods based on baseline wander estimating, methods based on high-pass filtering and method based on nonlinear filtering [3, 9, 1, 7].

This paper presents a new digital filter design method motivated by the baseline wander reduction problem. In order to eliminate, as much as possible, the noise from the processed signal but without distorting the useful part, there is a growing need for flexible digital filter design techniques that accept sophisticated specifications. This paper describes a new digital filter design method that uses not only the constraints on the designed filter's frequency response but takes also into account the constraints on the output, time domain, signal. This approach exploits the ε-insensitive loss function, that plays recently an important role in a vast range of intelligent processing systems, e.g. [10, 4, 5, 6]. Selected performances of the resulting filter are presented with respect to filter proposed in [12].

2. NEW CONSTRAINED FILTER DESIGN METHOD

Many recent digital filter design methods aim at minimizing a given error criterion. For formulating the design problems it is useful to define an error function

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$$E_c(\omega) = H(e^{j\omega}) - D(e^{j\omega}) \tag{1}$$

where $H(e^{j\omega})$ and $D(e^{j\omega})$ are the actual and the desired frequency response of the filter, respectively. If the impulse response $h(n)$ of the FIR filter has even or odd symmetry the phase response of the designed filter is linear and the resulting design problem is real-valued. In this case, the actual frequency response function $H(e^{j\omega})$ can be replaced with a real-valued function $H_0(e^{j\omega})$, called amplitude response, and related to the actual frequency response function by the following equation

$$H(e^{j\omega}) = e^{-j(N-1)/2\omega} e^{-j\beta} H_0(\omega) \tag{2}$$

where $\beta \in \{0, \pi/2\}$, and the desired frequency response of the filter, $D(e^{j\omega})$, is replaced with $D_0(e^{j\omega})$.

The constrained digital filter design problem can be view as a solution of a linear regression model described by

$$\mathbf{Y} = \frac{1}{2}\mathbf{X}\cdot\mathbf{b}' + \mathbf{X_0}\cdot b_0, \tag{3}$$

and

$$\mathbf{D_0} = \mathbf{T}\mathbf{b}, \quad \mathbf{b} \in \mathbb{R}^{M+1} \tag{4}$$

where $\mathbf{D}_0$ specifies the desired amplitude response d_i at frequency point f_i, L is a number of frequency points, $x(n)$ and $y(n)$ are, respectively, the input and output sequence, M is the filter order, and the following definition are used:

$$\mathbf{D_0} = [d_1, d_2, \ldots, d_L]^T, \mathbf{b}' = [b_1, \ldots, b_M]^T, \mathbf{b} = [b_0,\ \mathbf{b}'^T]^T,$$

$$\mathbf{T} = \begin{bmatrix} \cos(0\cdot\omega_1) & \cos(\omega_1) & \cdots & \cos(M\cdot\omega_1) \\ \cos(0\cdot\omega_2) & \cos(\omega_2) & \cdots & \cos(M\cdot\omega_2) \\ \vdots & \vdots & \ddots & \vdots \\ \cos(0\cdot\omega_L) & \cos(\omega_L) & \cdots & \cos(M\cdot\omega_L) \end{bmatrix}.$$

$$\mathbf{X_0} = [x(M+1), \ldots, x(K-M)]^T = [x_1, \ldots, x_{K-2M}]^T,$$

$$\mathbf{Y} = [y(2M+1), \ldots, y(K)]^T = [y_1, \ldots, y_{K-2M}]^T,\ \mathbf{X} = [\mathbf{x}_1^T, \ldots, \mathbf{x}_{K-2M}^T] = \mathbf{S_1} + \mathbf{S_2},$$

$$\mathbf{S_1} = \begin{bmatrix} x(M+2) & \cdots\ x(2M+1) \\ \vdots & \ddots\quad \vdots \\ x(K-M+1) & \cdots\quad x(K) \end{bmatrix}, \mathbf{S_2} = \begin{bmatrix} x(M) & \cdots & x(1) \\ \vdots & \ddots & \vdots \\ x(K-M-1) & \cdots & x(K-2M) \end{bmatrix}.$$

Using the ε-insensitive loss function, introduced by Vapnik in [11], and defined for a vector argument as

$$\rceil \mathbf{g} \lceil_\varepsilon \triangleq \sum_{i=1}^{L} \rceil g_i \lceil_\varepsilon \tag{5}$$

$$\rceil g_i \lceil_\varepsilon \triangleq \begin{cases} 0, & |g_i| \le \varepsilon, \\ |g_i| - \varepsilon, & |g_i| > \varepsilon. \end{cases} \tag{6}$$

the parameters vector $\mathbf{b}$ is obtained as a solution of the following problem

$$\underset{\mathbf{b}\in\mathbb{R}^{M+1}}{\text{minimize}}\ E_{\varepsilon,\delta}(\mathbf{b}) \triangleq \frac{1}{2}\mathbf{b}^T\mathbf{b} + C\sum_{i=1}^{L} \rceil d_i - \mathbf{t}_i\mathbf{b} \lceil_\varepsilon + V\sum_{k=1}^{K} \rceil y_k - \mathbf{x}_k\mathbf{b} \lceil_\delta, \tag{7}$$

where the parameters $C, V > 0$ controls the trade-off between the coefficient energy and the amount of frequency and time domains bound errors. To cooperate with (possibly) infeasible constraints, the preceding criterion can be written (for all impulse response points d_i, input x_k and output y_k), in the following (primal) form

$$\underset{\mathbf{b},\xi_i^+,\xi_i^-,\mu_k^+,\mu_k^-}{\text{minimize}} \ \tfrac{1}{2}\mathbf{b}^T\mathbf{b} + C\sum_{i=1}^{L}\left(\xi_i^+ + \xi_i^-\right) + V\sum_{k=1}^{K}\left(\mu_k^+ + \mu_k^-\right),$$

$$\text{subject to} \begin{cases} d_i - \mathbf{t}_i\mathbf{b}' - b_0 \le \varepsilon_i + \xi_i^+, \\ \mathbf{t}_i\mathbf{b}' + b_0 - d_i \le \varepsilon_i + \xi_i^- \\ \xi_i^+ \ge 0,\ \xi_i^- \ge 0 \\ y_k - \mathbf{x}_k\mathbf{b}' - x_k b_0 \le \delta_k + \mu_k^+, \\ \mathbf{x}_k\mathbf{b}' + x_k b_0 - y_k \le \delta_k + \mu_k^- \\ \mu_k^+ \ge 0,\ \mu_k^- \ge 0 \end{cases} \tag{8}$$

where $i = 1,\ldots,L$, $k = 1,\ldots,K$. The Lagrangian function of the preceding criterion is given by

$$\begin{aligned} \mathscr{L} = {} & \frac{1}{2}\mathbf{b}^T\mathbf{b} + C\sum_{i=1}^{L}\left(\xi_i^+ + \xi_i^-\right) + V\sum_{k=1}^{K}\left(\mu_k^+ + \mu_k^-\right) + \\ & -\sum_{i=1}^{L}\alpha_i^+\left(\varepsilon_i + \xi_i^+ - d_i + \mathbf{t}_i\mathbf{b}' + b_0\right) - \sum_{i=1}^{L}\alpha_i^-\left(\varepsilon_i + \xi_i^- + d_i - \mathbf{t}_i\mathbf{b}' - b_0\right) \\ & -\sum_{k=1}^{K}\beta_i^+\left(\delta_i + \mu_k^+ - y_k + \mathbf{x}_k\mathbf{b}' + x_k b_0\right) - \sum_{i=1}^{L}\left(\eta_i^+\xi_i^+ + \eta_i^-\xi_i^-\right) \\ & -\sum_{k=1}^{K}\beta_i^-\left(\delta_i + \mu_k^- + y_k - \mathbf{x}_k\mathbf{b}' - x_k b_0\right) - \sum_{k=1}^{K}\left(\lambda_k^+\mu_k^+ + \lambda_k^-\mu_k^-\right). \end{aligned} \tag{9}$$

The dual optimization problem of (8) we get by setting the derivatives (with respect to the primal variables) of (9) equal to zero

$$\begin{cases} \frac{\partial\mathscr{L}}{\partial\mathbf{b}} = \mathbf{b} - \sum_{i=1}^{L}\left(\alpha_i^+ - \alpha_i^-\right)\mathbf{t}_i - \sum_{k=1}^{K}\left(\beta_k^+ - \beta_k^-\right)\mathbf{x}_k = \mathbf{0}, \\ \frac{\partial\mathscr{L}}{\partial b_0} = \sum_{i=1}^{L}\left(\alpha_i^+ - \alpha_i^-\right) + \sum_{k=1}^{K}\left(\beta_k^+ - \beta_k^-\right)x_k = 0, \\ \frac{\partial\mathscr{L}}{\partial\xi_i^+} = C - \alpha_i^+ - \eta_i^+ = 0, \\ \frac{\partial\mathscr{L}}{\partial\xi_i^-} = C - \alpha_i^- - \eta_i^- = 0, \\ \frac{\partial\mathscr{L}}{\partial\mu_k^+} = V - \beta_k^+ - \lambda_k^+ = 0, \\ \frac{\partial\mathscr{L}}{\partial\mu_k^-} = V - \beta_k^- - \lambda_k^- = 0. \end{cases} \tag{10}$$

Substituting (10) into the Lagrangian (9), results in the following optimization problem

$$\underset{\alpha_i^+,\alpha_i^-,\beta_k^+,\beta_k^-}{\text{maximize}} \ \mathscr{L}$$

$$\text{subject to} \begin{cases} \sum_{i=1}^{L}\left(\alpha_i^+ - \alpha_i^-\right) + \sum_{k=1}^{K}\left(\beta_k^+ - \beta_k^-\right)x_k = 0, \\ 0 \le \alpha_i^+, \alpha_i^- \le C, \\ 0 \le \beta_k^+, \beta_k^- \le V. \end{cases} \tag{11}$$

where the Lagrangian is given by the expression

$$\mathscr{L} = -\frac{1}{2}\Big(\sum_{i=1}^{L}\left(\alpha_i^+ - \alpha_i^-\right)\mathbf{t}_i + \sum_{k=1}^{K}\left(\beta_k^+ - \beta_k^-\right)\mathbf{x}_k\Big)\cdot$$
$$\Big(\sum_{i=1}^{L}\left(\alpha_i^+ - \alpha_i^-\right)\mathbf{t}_i + \sum_{k=1}^{K}\left(\beta_k^+ - \beta_k^-\right)\mathbf{x}_k\Big)^T - \sum_{i=1}^{L}\left(\alpha_i^+ + \alpha_i^-\right)\varepsilon_i \tag{12}$$
$$+ \sum_{i=1}^{L}\left(\alpha_i^+ - \alpha_i^-\right)d_i - \sum_{k=1}^{K}\left(\beta_k^+ + \beta_k^-\right)\delta_k + \sum_{k=1}^{K}\left(\beta_k^+ - \beta_k^-\right)y_k.$$

At the saddle point the Karush-Kühn-Tucker (KKT) conditions must be satisfied

$$\begin{cases} \alpha_i^+\big(\varepsilon_i + \xi_i^+ - d_i + \mathbf{t}_i\mathbf{b}' + b_0\big) = 0, \\ \alpha_i^-\big(\varepsilon_i + \xi_i^- + d_i - \mathbf{t}_i\mathbf{b}' - b_0\big) = 0, \\ \beta_k^+\big(\delta_k + \mu_k^+ - y_k + \mathbf{x}_k\mathbf{b}' + x_k b_0\big) = 0, \\ \beta_k^-\big(\delta_k + \mu_k^- + y_k - \mathbf{x}_k\mathbf{b}' - x_k b_0\big) = 0, \\ (C - \alpha_i^+)\xi_i^+ = 0, \\ (C - \alpha_i^-)\xi_i^- = 0, \\ (V - \beta_k^+)\mu_k^+ = 0, \\ (V - \beta_k^-)\mu_k^- = 0. \end{cases} \tag{13}$$

The parameters $\mathbf{b}'$ could be determined form the first condition of (10)

$$\mathbf{b}' = \mathbf{t}^T\left(\alpha^+ - \alpha^-\right) + \mathbf{x}^T\left(\beta^+ - \beta^-\right), \tag{14}$$

and the parameter b_0 can be determined from the following equations, derived from KKT conditions (13), by taking arbitrary condition for which the corresponding condition is met

$$b_0 = \begin{cases} d_i - \mathbf{t}_i\mathbf{b}' - \varepsilon_i, & \text{for } 0 < \alpha_i^+ < C, \\ d_i - \mathbf{t}_i\mathbf{b}' + \varepsilon_i, & \text{for } 0 < \alpha_i^- < C, \\ \big(y_k - \mathbf{x}_k\mathbf{b}' - \delta_k\big)/x_k, & \text{for } 0 < \beta_k^+ < V, \\ \big(y_k - \mathbf{x}_k\mathbf{b}' + \delta_k\big)/x_k, & \text{for } 0 < \beta_k^- < V. \end{cases} \tag{15}$$

3. EXPERIMENTAL RESULTS

The usefulness of the proposed filter design method were investigated using some standards developed by the International Electrotechnical Commission (IEC) within the European project “Common Standards for Quantitative Electrocardiography” in order to test the accuracy of ECG signal processing methods. These Common Standards are very well suited to analyze ECG system’s software performance in term of baseline removal, line frequency suppression, waveform detection, localization of fiducial points, measurement of ECG parameters, etc. They establish also exact procedures for testing hardware aspects of ECG systems, for example, calibration, amplifier linearity, gain factors, etc.

As a possible approach to evaluate methods used for ECG baseline wander reduction, the IEC committee suggests artificial signal composed of triangular waves. The triangular wave is 1.5 mV high and has 80 ms base width. This signal shall not produce an output signal with an offset from the isoelectric line greater than 20 μV , and shall not produce a slope greater than 50 μV/s in a 200 ms region following the impulse and a slope of 100 μV/s anywhere outside the region of the impulse. On the other hand, the amplitude response of the required low-pass filter in the 0.67 – 40 Hz passband, should not have ripples greater than ±10%. In other frequency bands the constraints are less important. The described specifications represents a highpass filter with a very narrow transition band and constraints in both frequency and time domains. Using classical filter design methods e.g. [8] it was impossible to find filter that fulfils all the constraints. Also baseline wander removing filter proposed in [12] does not met the given above requirements (see Fig. 1).

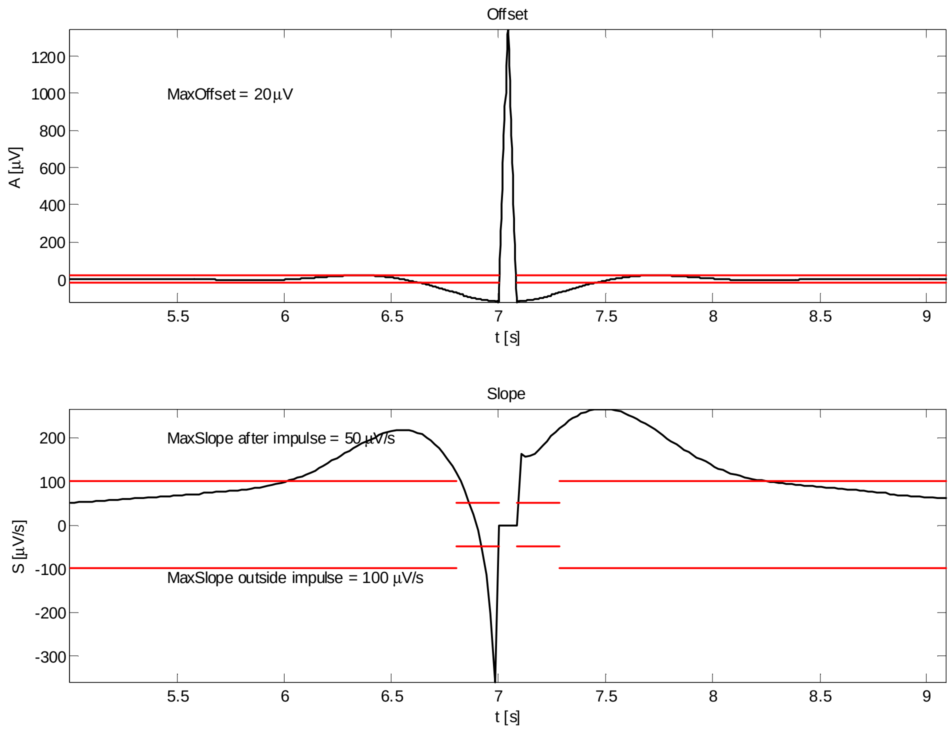

Fig. 1. Time domain response of the filter presented in [12] for a triangular-wave signal.

The proposed filter design method was used to calculate the required filter coefficients. The minimum filter order, sufficient to fulfil the specified constraints was found to be equal 890. The parameters $\varepsilon_i,\ i=1,\ldots,L,\ \ \delta_k,\ k=1,\ldots,K$ was chosen according to the frequency and time domain constraints, defined above. Fig. 2 presents the time domain response of the designed filter applied to the described above triangular-wave signal. All constraints concerning the maximum distortion and slope were fulfilled. Also, the frequency response of the filter, not shown here, does not exceed the predefined limits.

4. CONCLUSIONS

In this paper a new method for digital filter design was presented. This method allows to define the filter's specification not only in the frequency domain, but also with respect to the required output signal. The possibilities offered by this new method was shortly illustrated with design of ECG highpass filter for baseline wander reduction.

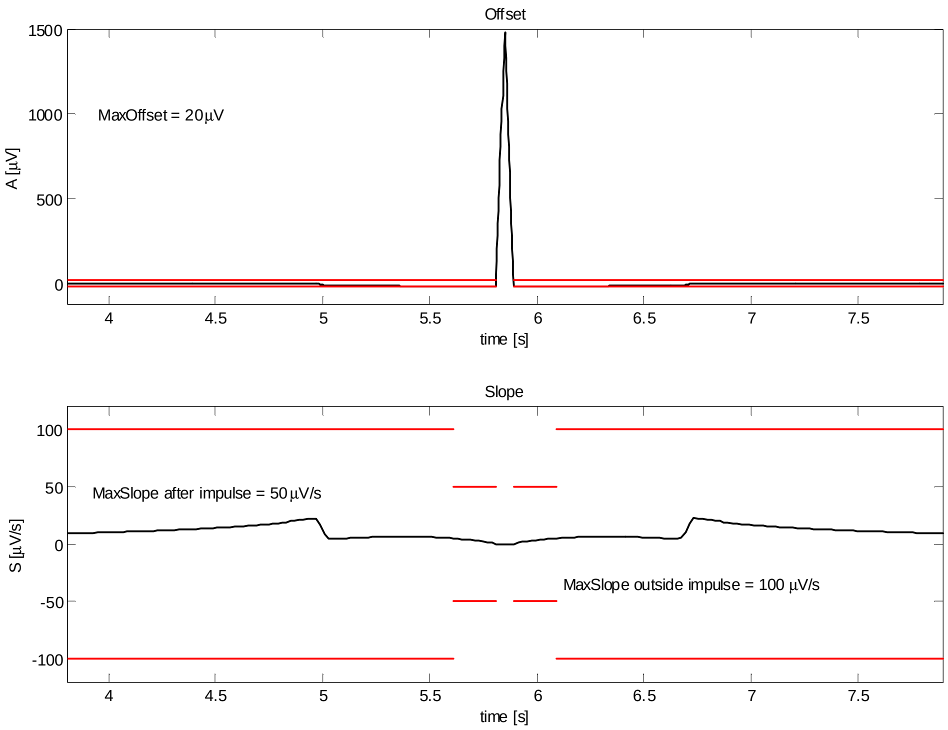

Fig. 2. Time domain response of the designed filter for a triangular-wave signal..

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