*State-space projections,
dynamic time warping, time averaging,
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APPLICATION OF DYNAMIC TIME WARPING TO ECG PROCESSING

The paper presents application of dynamic time warping (DTW) to ECG noise suppression. The operation (DTW) was introduced to the traditional technique of the synchronized time averaging and allowed to overcome the averaging inability to preserve the desired component morphological variability. The developed technique was investigated and compared to other modern techniques of ECG signal enhancement. Its impact on the precision of repolarization duration measurements is presented.

1. INTRODUCTION

The noninvasive electrocardiology has its roots in the late 19th century. Subsequently, this new field was intensively developed. A significant progress in ECG signals analysis was achieved by application of digital filters with linear phase response allowing for suppression of ECG noise with limited distortions of the desired component. This type of filters is particularly effective when applied to baseline wander and powerline interference suppression. Unfortunately, wide frequency band of the electromyographic (emg) noise (overlapping the frequency band of the desired ECG) makes them practically useless. To mitigate the problem synchronized time averaging was introduced. The technique performs time-alignment of ECG beats and construction of an average template. As a result the uncorrelated noise is suppressed and the signal-to-noise ratio is raised. However, variability of ECG morphology gets suppressed as well. Investigations performed in 1990s (QT dispersion, T wave alternans, and various others) resulted in the necessity to device new techniques, aimed at ECG enhancement without reducing its morphological variability. The increasing computational power of modern computers caused a great progress in ECG signal analysis, achievable by application of the methods from the field of nonlinear dynamics, such as the method of nonlinear state-space projections [1] also called as projective filtering. A modification of the method, based on introduction of ECG beats time-alignment [2], resulted in high reduction of the method’s computational costs while increasing its robustness against high energy emg noise. Further increase of projective filtering performance was achieved by introduction of nonlinear alignment of ECG beats [3]. The operation of nonlinear alignment is realized by application of dynamic time warping [4]. The present study shows that not only projective filtering but also more traditional techniques of ECG processing, such as time averaging can be improved by application of dynamic time warping.

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2. METHODS

2.1. NONLINEAR STATE-SPACE PROJECTIONS (NSSP)

The method performs reconstruction of the state-space representation of the observed signal by application of Takens embedding operation, where a point in the constructed space is a vector:

$$\mathrm{x}^{(n)} = \left[x(n-\tfrac{m}{2}\tau), x(n-(\tfrac{m}{2}-1)\tau), \ldots, x(n+(\tfrac{m}{2})\tau)\right] \tag{1}$$

x(n) is the processed signal, τ is the time lag (τ=1 in this study), *m* is the embedding dimension. To simplify further considerations the equation defining the embedding operation was written in a symmetric form (the sample *x(n)* is the central coordinate of the vector $\mathbf{x}^{(n)}$).

For each point $\mathbf{x}^{(n)}$ a small neighborhood is constructed, composed of the points which are close to $\mathbf{x}^{(n)}$. Within each neighborhood a local mean is computed and a covariance matrix of the deviations from the mean. Then the principal subspace [1] of these deviations is constructed (of the assumed dimension) and the point $\mathbf{x}^{(n)}$ is projected into this subspace. Then one-dimensional representation of the signal *x(n)* is reconstructed.

Performance of NSSP is highly dependent on the results of neighborhood determination. In nonstationary electromyographic noise the neighborhood determination errors limit the method's performance. A modification was introduced in [2] to overcome this problem: instead of the Euclidean distance between points their position within an ECG beat was employed as a criterion for neighborhood determination. With such a criterion the trajectory points are projected into the subspaces corresponding to their positions within ECG beats. The developed method was called projective filtering of time-aligned beats (PFTAB).

2.2. DYNAMIC TIME WARPING (DTW)

Lets consider a classical approach to nonlinear alignment (time warping) of the time series $x(n)$, n = 0, 1, 2, ..., N_x and $y(n)$, n = 0, 1, 2, ..., N_y. To perform this operation we calculate the matrix of costs $\mathbf{D}=[d_{i,j}]$, i=0, 1, …, N_x, j=0, 1, …, N_y whose entries are equal

$$d_{i,j} = \left(x(i) - y(j)\right)^2 . \tag{2}$$

Each entry $d_{i,j}$ corresponds to the alignment of $x(i)$ and $y(j)$. The warping path of the series $x(n)$ and $y(n)$ consists of the pairs (i_l, j_l) that indicate the successive aligned elements $x(i_l)$, $y(j_l)$, l=0, 1, ..., L. In the classical approach the warping path is subject to the following constraints.

Boundary conditions: $i_0 = j_0 = 0$, $i_L=N_x$, $j_L=N_y$, which force the warping path to start with the pair $x(0)$, $y(0)$ and to end with $x(N_x)$, $y(N_y)$.
Continuity conditions: $i_{l+1}-i_l \le 1$, $j_{l+1}-j_l \le 1$, which prevent the elements of both series from being omitted in the warping path.
Monotonicity conditions: $i_{l+1} \ge i_l$, $j_{l+1} \ge j_l$, which force the elements of both series to occur in the warping path in a non-decreasing order.

The warping path that minimizes the total cost of the alignment

$$Q = \sum_{l=0}^{L} d_{i_l, j_l} \ . \tag{3}$$

is searched for. It can be found with the use of dynamic programming [4]. First a matrix of cumulative costs is constructed, according to the recursive definition

$$g_{i,j} = d_{i,j} + \min \{ g_{i-1,j-1}, g_{i-1,j}, g_{i,j-1} \}. \tag{4}$$

where $g_{0,0} = d_{0,0}$, and $g_{-1,j} = \infty$, $g_{i,-1} = \infty$ for all i,j (this definition has its pictorial representation which shows the allowed step-directions of the warping path, see Fig.1.A).

Then, starting from the upper right corner of **G** ($i=N_x$, $j=N_y$), the warping path is searched for by backtracking along the allowed step-directions through the minima of the cumulative costs in **G**, until i=0, j=0. While backtracking the allowed step-directions are opposite to that presented in Fig.1.A.

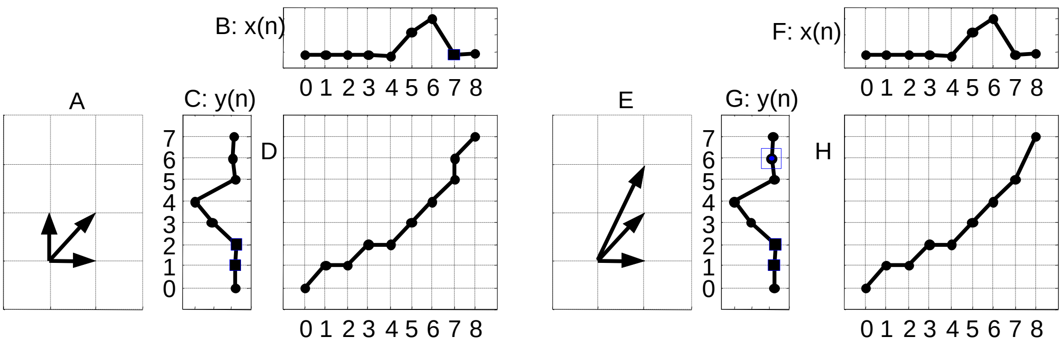

Fig.1. Dynamic time warping of time series. A,E) Pictorial representations of the definitions (4) and (5). B,C,E,F) Time-warped series:■ marks the elements that occur twice, whereas □ the elements that are omitted in the warping paths. D,H) Graphical representations of two warping paths: {(0,0),(1,1),(2,1),(3,2),(4,2),(5,3),(6,4),(7,5),(7,6),(8,7)} and {(0,0),(1,1),(2,1),(3,2),(4,2),(5,3),(6,4),(7,5),(8,7)} respectively.

Applied to time warping of the series *x(n)* and *y(n)* this classical approach produced the warping path presented in Fig.1.D. We can notice that, to achieve nonlinear alignment of both series, two elements of *y(n)* and one element of *x(n)* had to occur two times in the warping path.

The presented approach to time warping can be modified by introducing a constraint $i_l = l$, $l = 0, 1, \dots, N_x$ and by substituting the continuity conditions with the condition restricting the number J of the successive elements of $y(n)$ that may be omitted in the warping path $j_{l+1}-j_l \leq J+1$. When J=1 only one of the successive two elements can be omitted - the constraints can be satisfied by the algorithm based on the following definition of the matrix of cumulative costs

$$g_{i,j} = d_{i,j} + \min \{ g_{i-1,j}, g_{i-1,j-1}, g_{i-1,j-2} \}. \tag{5}$$

where $g_{0,0} = d_{0,0}$, and $g_{-1,j} = \infty$, $g_{i,-1} = \infty$, $g_{i,-2} = \infty$ for all i,j. This definition has its pictorial representation in Fig.1.E.

Application of these conditions to time warping of the series $x(n)$ and $y(n)$ produced the warping path presented in Fig.1.H. We can notice that all elements of the series $x(n)$ occurred only once in the warping path. Instead of the second occurrence of the element $x(7)$ (like as in D) the element $y(6)$ was omitted. We can conclude that the series y(n) was nonlinearly aligned with respect to the series x(n), whose time axis was unchanged.

The technique can be adapted to the alignment of sequences of points of the state space trajectory (eq.1). To this end the elements of the matrix of costs are defined as

$$d_{i,j} = \left\| \mathrm{x}^{(i)} - \mathrm{y}^{(j)} \right\|^2 . \tag{6}$$

2.3. TIME WARPING ECG SIGNALS

Detection and synchronization of QRS complexes produces a set of fiducial points $\{r_i : i = 1, 2, ..., I\}$ corresponding to the same position within the respective I complexes. The operation of nonlinear alignment will be applied to ECG sections starting b samples before r_i and ending b samples after r_{i+1} : $r_i - b \le n \le r_{i+1} + b$, where b is a small number. Aligning such sections of ECG signal we do not need to decide where individual beats begin and end (which is sometimes not a trivial task). In fig.2 we can notice that application of the matrix of costs defined by (6) (time warping the sequences of points instead of one-dimensional time series) gives better results.

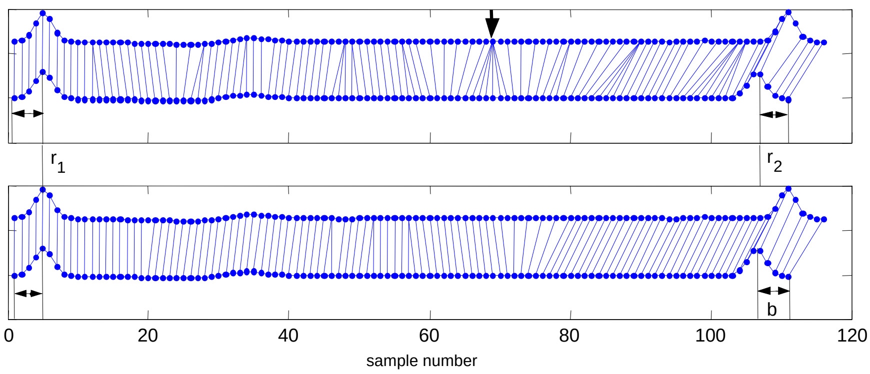

Fig.2. The results of ECG signal time warping when the matrix of costs is defined by eq.2 (A) and by eq.6 (B). The lower signals in both panels have the time axis unchanged, whereas the upper ones are time warped (the aligned samples are connected by solid lines). The arrow indicates one of the signal samples, which occur many times in the warping path. The applied sampling frequency is 62.5 Hz.

The warping path is more regular. When definition (2) is applied we can notice that quite often individual samples of the upper signal appear many times in the warping path, whereas many of the neighbouring samples are omitted (the signals were decimated to make this effect visible).

2.4. STATE-SPACE AVERAGING (SSA)

Application of definition (6) to the nonlinear alignment of the respective I-1 sequences of state-space points results in the construction of neighbourhoods corresponding to the positions within the aligned ECG sections. Lets denote the points occupying a single l-th position within the aligned sections of ECG signal as $\Gamma^{(l)}$. In order to suppress noise while preserving the morphological variability of the desired ECG we propose a kind of selective averaging. For each point $\mathbf{x}^{(n)} \in \Gamma^{(l)}$ we search for the smaller neighbourhood containing the assumed fraction (50% in this study) of the nearest points (among the points contained in $\Gamma^{(l)}$) and we replace $\mathbf{x}^{(n)}$ by the average of these points. Then we reconstruct one-dimensional representation of the signal.

This operation (SSA) is related to time averaging but it does not lead to complete suppression of the desired signal morphological variability.

3. RESULTS

Qualitative results of ECG enhancement by application of time averaging or state-space averaging are presented in Fig.3. Both methods managed to suppress noise effectively. When, however, the position of the T wave was slightly different than the average one TA method failed to preserve this information and produced high waves in the residual noise. SSA method managed to preserve the information on dynamically changing T wave position and the residual noise contains only negligible components. Quantitative results are presented in Table.1. In the experiment performed 5 signals of high quality from the MIT-BIH database were chosen to simulate the desired ECG. The signals were contaminated additively with the emg noise (stored in the MIT-BIH database as well). Then the simulated noisy ECGs were enhanced by application of NSSP, PFTAB

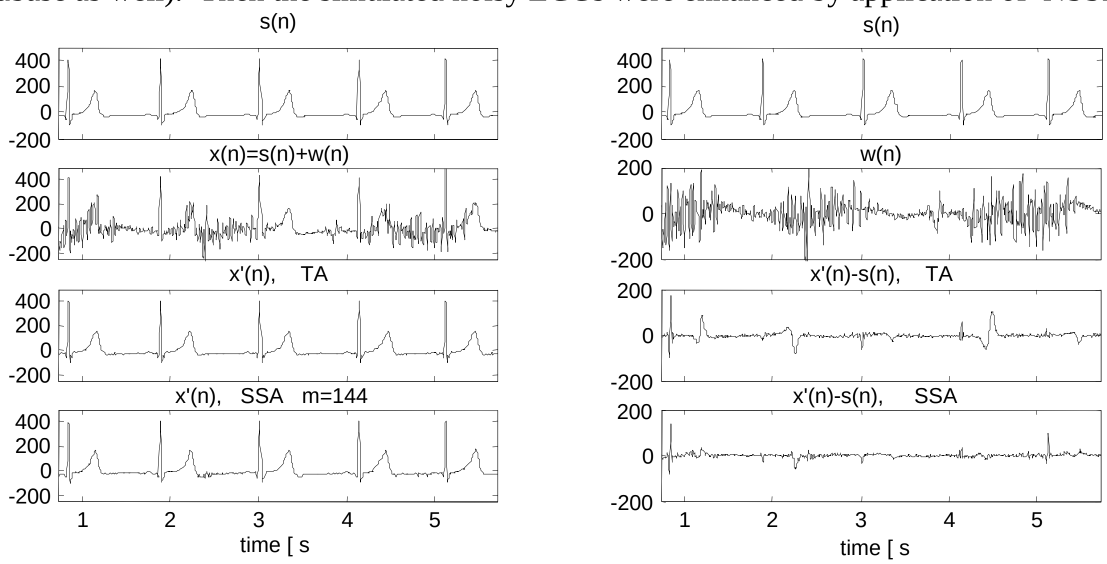

Fig.3. Visual results of ECG enhancement by application of either time averaging (TA) or state-space averaging (SSA): s(n) – the signal with variable position of the T wave, w(n) – the emg noise, x(n) – the additively contaminated signal, x'(n) – the enhanced signal, x'(n)-s(n) – the residual noise.

and SSA methods. The results obtained were estimated with the use of the following noise reduction factor

$$NRF = \sqrt{\sum_n \left(x(n) - s(n)\right)^2 \Big/ \sum_n \left(x'(n) - s(n)\right)^2} \, . \qquad (7)$$

where $s(n)$ is the desired ECG, $x(n)$-$s(n)$=$w(n)$ is the noise (added), $x'(n)$ is the enhanced signal, $x'(n)$-$s(n)$ is the residual noise.

Table.1. The average *NRF* obtained in the respective parts of ECG beats.

SNR [dB]	5	10	20	5	10	20	5	10	20	5	10	20
Method	Overall			P wave			Q wave			T wave		
NSSP	2.34	2.27	1.31	2.57	2.40	1.82	1.82	2.17	0.71	2.44	2.27	1.78
PFTAB	3.14	2.61	1.42	5.38	3.62	1.43	1.73	1.61	1.20	4.04	2.93	1.59
SSA	3.33	2.37	0.97	5.75	4.01	1.47	1.88	1.25	0.49	3.89	3.03	1.44

We can notice that for low level of noise the SSA method is less effective than NSSP and PFTAB. With the growing level of noise, however, its performance is more and more competitive.

The influence of SSA on the measurement of the repolarization duration [5] (RT_{max} interval in this study) is presented in Fig.4. The SSA preprocessing limited the number of large measurement errors of the algorithm applied.

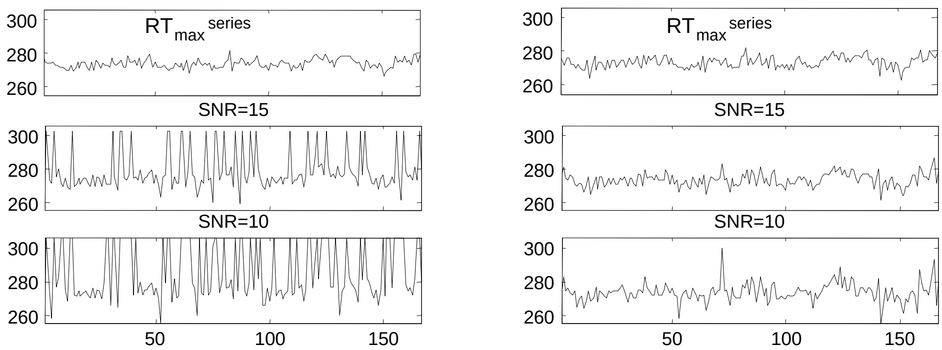

Fig.4. The results of RT_{max} series measurement without (on the left) and with SSA preprocessing (on the right). The uppermost series were obtained for the signals of high quality from the QT database. Below are the series obtained for the signals contaminated artifficially with emg of two different levels (the average SNR is displayed in the figure). When a measured value exceeded a threshold (as a result of large measurement error) it was truncated.

4. CONCLUSION

ECG signals, although approximately repeatable, contain important information in their slight morphological variability. An example of such a phenomenon is a shifting T wave. The traditional technique of ECG enhancement based on the synchronized time averaging cannot handle this problem. Application of the state-space embedding, nonlinear alignment of the state-space vectors, and the search for the neighbourhoods of the aligned vectors were the operations which helped to improve time averaging. The modified technique appeared to deal successfully with the problem of ECG enhancement prior to the measurements of the dynamically changing repolarization duration.

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